Table 1 **Initial and final solutions of the gains**

Control gains	FDPC (IS)	FDPC (FS)	LO (IS)	LO (FS)
$K_{z_{11}}$	$.109\times10^{-2}$	$.104\times10^{-2}$	$.134\times10^{-1}$	$.134\times10^{-1}$
$K_{z_{21}}$	$.589\times10^{-4}$	$.602\times10^{-3}$	$.245\times10^{-1}$	$.245\times10^{-1}$
$K_{x_{11}}$	$.134\times10^{-1}$	$.160\times10^{-1}$	–	–
$K_{x_{21}}$	$.245\times10^{-1}$	$.841\times10^{-1}$	–	–
$C_{d_{11}}$	$.100\times10^{3}$	$.128\times10^{3}$	$.100\times10^{2}$	$.594\times10^{2}$
$C_{d_{12}}$	$.778\times10^{3}$	$.799\times10^{3}$	$.244\times10^{3}$	$.217\times10^{3}$
$C_{e_{11}}$	$.100\times10^{2}$	$-.565\times10^{2}$	–	–
$C_{e_{12}}$	$.244\times10^{3}$	$.186\times10^{3}$	–	–

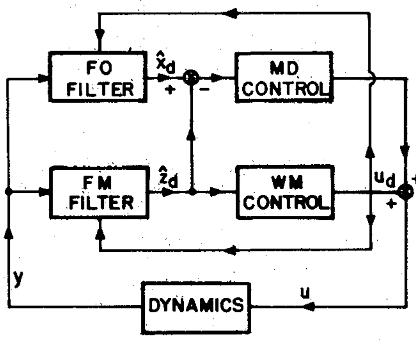

Fig. 1 **FDPC controller.**

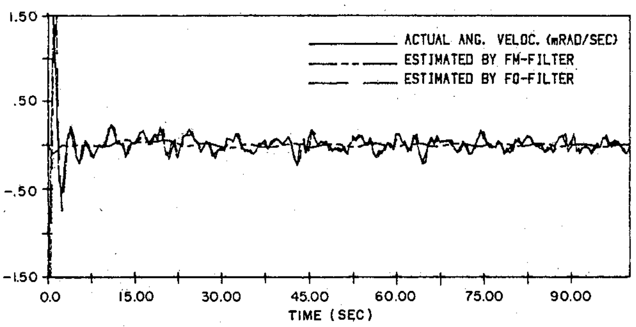

Fig. 2 **Actual and estimated angular velocity (FDPC controller).**

15, 17, and 19 are given in Ref. 4. By adoption of a second-order work model, the FDPC controller obtained is fourth-order, while the LO is second-order. This one is used here for comparisons because it is less sensitive to vibration frequency variations and to high-order vibration modes.[4-5] References 4-5 show the sensitivity analysis and simulation results for second and higher-order controllers.

The initial solutions (IS) and final solutions (FS) for the FDPC and LO controllers are presented in Table 1. The initial stable solutions were taken from Ref. 3. The weighing parameters for the performance index [Eq. (5)] were chosen according to the response in angular drift and velocity (A_1), actuator requirements (B), filters behavior (A_3,A_4) and response of the truncated modes (A_2). A_1 was assumed diagonal with the two nonnull elements equal to 1×10^4; A_2,A_3 were assumed null matrices; A_4 was assumed identity matrix; and the parameter B was taken equal to one.

The sensitivity to the variation of the first modal frequency was tested by changing it by 20%. Both controllers have shown to be low-sensitive. Simulations using digital integrations were performed for the fourteenth-order evaluation model. A control pulse $u=20.$ during .02 s sets the initial conditions. For the evaluation model, the performance of the LO controller became degraded, especially when the ninth vibration mode was introduced. However, the FDPC controller performance was not degraded by the high-order vibration modes.

The actual and estimated angular velocity responses (see Fig. 2) illustrate the results for the FDPC controller. The objectives proposed by the heuristic gain determination procedure[1-3] were achieved. The FO-estimates are close to the actual coordinates, and the FM-estimates do not seem to include the influence of the truncated coordinates.

Conclusions

The presented double-path compensating controller satisfies the requirements of performance, sensitivity to modeling errors, and simplicity for on board implementation. This structure induces the system to follow the faithful model filter in the same fashion as the model reference adaptive scheme induces the system to follow the reference model. This analogy could be explored in future works by applying the model reference algorithms to the proposed structure.

References

[1]Ceballos, D.C., "Controlador com técnica de compensacão baseada em um esquema de controle ativo sobre o efeito acumulado do desvio da modelagem," Doctoral Thesis, Instituto de Pesquisas Espaciais, São José dos Campos, Brazil, 1983.

[2]Ceballos, D.C. and Rios Neto, A., "Técnica de compensacão utilizando um esquema de controle ativo para anular o efeito acumulado do desvio da modelagem," *Proceedings of the Brazilian Congress of Automatica,* Brazilian Society of Automatica, Vol. 1, 1984, pp. 58-76.

[3]Ceballos, D.C. and Rios Neto, A., "Truncated Elastic modes Coupling Effects Minimization Method by Using Attitude Control Scheme," *Proceedings of the Symposium on Modal Analysis,* Union College, Vol. 3, 1985, pp. 1227-1233.

[4]Larson, V. and Likins, P.W., "Optimal Estimation and Control of Elastic Spacecraft," *Control and Dynamics Systems,* Vol. 13 Academic, New York, 1977, pp. 285-322.

[5]Martin, G.D. and Bryson, Jr., A.E., "Attitude Control of a Flexible Spacecraft," *Journal of Guidance and Control,* Vol. 3, 1980, pp. 37-41.

[6]Kwakernaak, K. and Sivan, R., *Linear Optimal Control Systems,* Wiley-Interscience, New York, 1982, pp. 427-436.

[7]Barraud, A.Y., "An Accelerated Process to Solve Riccati Equation Via Matrix Sign Functions," *Proceedings of the 1979 IFAC Symposium on Computer Aided Design of Control Systems,* Pergamon, Oxford, UK, 1980, pp. 9-14.

Reduced-Order Observers Applied to State and Parameter Estimation of Hydromechanical Servoactuators

Hagop V. Panossian*
HR Textron, Valencia, California

I. Introduction

IDENTIFICATION and state and parameter estimation techniques can be utilized in fluid power control systems. The states and various critical parameters of a closed-loop actuation system can be estimated using modern control theory. Thus, the velocity, position, and chamber pressures can be

Received May 13, 1985; revision received Sept. 25, 1985.

*Engineering Supervisor.

estimated assuming realistic measurement information. Moreover, parameters like leakage can be estimated using the abovementioned estimates.

Knowledge or information regarding the states and parameters of an actuator can be utilized to enhance the performance or to detect anomalies of the system. Performance enhancement and detection of anomalous behavior can utlimately lead to solutions for backup provisions of critical control systems. Supersonic aircraft have inherent nonlinear characteristics due to high performance requirements. Thus, loss of operation of an actuator for a critical control surface, even for a very short period of time, can lead to catastrophic consequences.[1]

The present analysis is designed to assess the applicability and usefulness of modern control theory in servoactuator control or in detecting, on-line, and isolating anomalous behavior of such systems. A survey of literature in the state-of-the-art fluid power control technology indicated that the study undertaken herein is unique in its approach and objectives. In a separate study an extended Kalman-type filter was utilized on a full-scale nonlinear model to generate state estimates under the assumption of direct noisy measurements of all states. The results of the abovementioned simulations were very encouraging and will be reported separately. The simulations performed herein were that of a reduced-order observer.[2]

Position measurements only were assumed available, and a third-order linear model of a hydromechanical servoactuator under a sinusoidal plus a constant load was utilized for the estimation.[3] Piston rod velocity and piston rod acceleration were estimated using the given measurement. Moreover, leakage across the piston was also estimated and the results were plotted simultaneously with the analytical value.

II. Problem Statement

Consider a general linear model of a hydromechanical servoactuator controlled by a spool/sleeve-type four-way servovalve given in the following transfer function form[4]:

$$
\begin{aligned}
X_p = {} & (K_q/A_p)X_v - (K_{ce}/A_p^2)\,[(1+(v_t/4\beta_e K_{ce})/s]F_L \\
& \div \{(v_t M_t/4\beta_e A_p^2)s^3 + [(K_{ce}M_t/A_p^2) + (B_p v_t/4\beta_e A_p^2)]s^2 \\
& + [1 + (B_p K_{ce}/A_p^2) + (K v_t/4\beta_e A_p^2)]s + (K_{ce}K/A_p^2)\} \qquad (1)
\end{aligned}
$$

where X_p = piston rod position, X_v = servovalve spool position, K_q = valve flow gain = .019 in.^{3}/s/in., A_p = piston area = 0.961 in.^{2}, K_{ce} = total flow pressure coefficient = .00178 in.^{3}/s/psi, v_t = total contained volume of both chambers = 5.19 in.^{3}, β_e = effective bulk modulus of the system = 120,000 psi; F_L = arbitrary sinusoidal load force on piston = .01294 lb/s^2/in., B_p = viscous damping coefficient of piston and load = 44.4 in./lb/s, K = load spring gradient = 100,000 lb/in., and s = Laplace operator.

In Eq. (1), $K_{ce} = K_c + C_{LK} + C_{ep/2}$, where K_c = valve flow pressure coefficient signifying leakage across the spool of the servovalve, C_{LK} = cross-port leakage coefficient of the piston representing leakage flow across the piston, and C_{ep} = external leakage coefficient indicating leakage to the atmosphere from around the piston rod. If we consider the continuity equations for the two chambers of a simple actuator, then the following can be derived[4]:

$$
\begin{aligned}
C_{LK} = {} & -(2A_pX_pV_{t0}s - 2A_pX_p^2V_{t0}s) \\
& - \frac{(V_{01}+A_pX_p)(V_{02}-A_pX_p)\dot{P}_L}{\beta_e} \\
& + (V_{02}-A_pX_p)C_dW(U+X_v)\frac{1}{\rho}(P_s-P_L) \\
& - (V_{01}+A_pX_p)C_{01}W(U-X_v)\frac{1}{\rho}(P_L+P_s)/V_tP_L \qquad (2)
\end{aligned}
$$

where $V_{01}=0.7$ in.^{3}, $V_{02}=0.3$ in.^{3}, $V_t=5.19$ in.^{3}, $V_{to}=V_{01}+V_{02}/V_r$ = total enclosed volume, W = area gradient of ports of servovalve $=0.37\pi$ at full stroke, V_{01} and V_{02} = dead volumes, C_d = discharge coefficient = 0.61, ρ = fluid density $=0.78\times10^{-4}$ lb/s^2/in.^{4}, P_s = supply pressure = 3000 psi, and U = underlap = .0001 in. Here

$$P_L = \frac{1}{A_p}(M_tX_ps^2 + B_pX_ps + KX_p + F_1)$$

Thus the differential equation for the leakage across the piston is[4]

$$\frac{dQ_{LK}}{dt} = C_{LC}\dot{P}_L \qquad (3)$$

Hence the state vector is $[X_p, \dot{X}_p, \ddot{X}_p, Q_{LK}]^T$.

The mathematical model of hydraulic servomechanisms is nonlinear, and it essentially relates the speed of the piston rod to the valve spool displacement and to the total load. The nonlinearities are related to sharp-edged orifice flow equations and to frictional and inertial forces. However, under conditions of constant amplitude ratios and no bandwidth and natural frequency problems, linear models are quite valid. Moreover, coulomb or viscous friction is also almost completely eliminated when the actuator has hydrostatic bearings.[3] The present study assumes appropriate conditions for linearization.

To compensate for variations in parameters due to tolerances and various other factors, it is possible to utilize adaptive observers that will generate realistic estimates of the states and parameters concerned.[6] Moreover, leakage across the piston is a significant parameter. Many actuation failures are a result of a broken seals and excessive leakage across the piston. Thus, estimating the leakage across the piston and utilizing the information as a failure detection threshold is conceivably a practical approach. This is what is accomplished herein.

The Observer

Let the state space, time domain representation of the system in Eq. (1) be given by

$$\dot{x}(t) = Fx(t) + Lu(t) \qquad (4)$$

With given initial condition $x(t_0)=x_0$, where $x(t)$ is the $n\sim$dimensional state vector consisting of (in this case) position, velocity, acceleration, and leakage. F is the state matrix $(n\times n)$, L is the control matrix $(m\times n)$, and $u(t)$ is the $m\sim$dimensional control vector. Let the measurement system be given by

$$z(t) = Hx(t) \qquad (5)$$

where $z(t)$ is the $p\sim$dimensional measurement vector (in this case, the position of the piston rod) and H the measurement matrix [here taken as $H=(1000)$]. The problem under consideration entails generation of estimates of the states from the single available measurement. Thus, following Gelb,[4] we introduce a $(n\text{-}p)\sim$dimensional vector $y(t)$ such that

$$y(t) = Tx(t) \qquad (6)$$

where

$$\left[\begin{array}{c} T \\ \hline H \end{array}\right] \text{ is nonsingular}$$

thus, $x(t)$ can now be expressed as

$$x(t) = Ay(t) + Bz(t) \qquad (7)$$

where

$$\begin{bmatrix} T \\ \hdashline H \end{bmatrix}^{-1} = [A \vdots B] \quad \text{and} \quad AT+BH=i$$

$$\begin{bmatrix} T \\ \hdashline H \end{bmatrix} [A \vdots B] = I$$

should be satisfied. Then the differential equation for the observer will be given by

$$\dot{\hat{y}}(t) = TFA\hat{y}(t) + TFBz(t) + TLu(t) \tag{8}$$

and the state estimates are generated by

$$\hat{x}(t) = A\hat{y}(t) + Bz(t) \tag{9}$$

The observer error has exponential convergence rate[2] and is a function of the choice of B. It can be shown that if the system is completely observable, the matrix B can be chosen to produce any desired response by selecting the eigenvalues properly. It is known (see Refs. 2 and 3) that A and T are not unique in the time-invariant actuation and that the admissible pair (A,T) defines an equivalent class in the class of observers with exponential error convergence rates. It should be noted that there is no rule that directs one in selecting the desired eigenvalue. One should consider the specific problem at hand and make a choice based on engineering intuition and practical considerations.

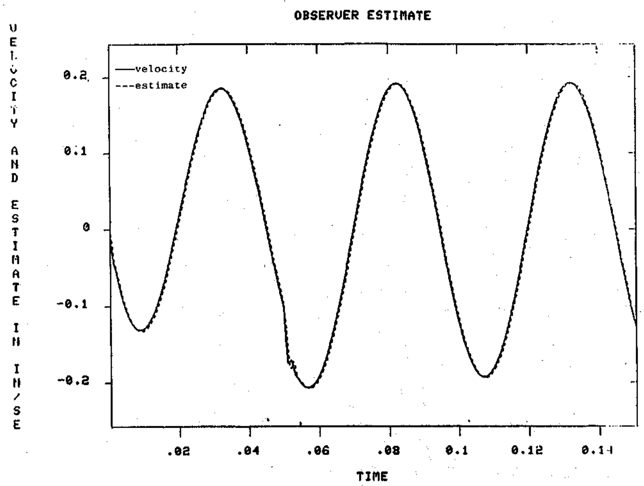

Fig. 1 Observer estimate of perturbation in velocity.

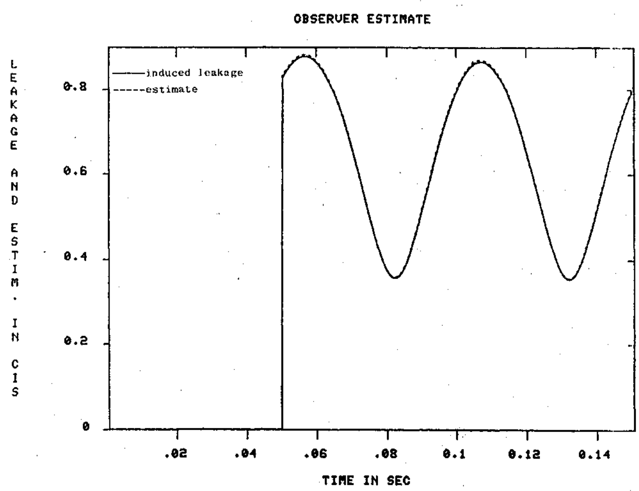

Fig. 2 Observer estimate of induced leakage.

III. Simulation and Results

The main objective of the simulations in the present study was to assess the applicability of observers for state and parameter estimation of hydromechanical servoactuation systems. The results are very promising. The system considered was generic, and its observability, especially for the reduced-order observer situation, proves to be practicably significant.

The velocity and acceleration of the actuator piston rod was estimated herein, as well as leakage across the piston. However, various other parameters, such as spool position of the servovalve, the load pressure, etc., can be estimated in a similar manner. The results are plotted in Figs. 1 and 2. A small leakage was introduced through a small opening on the piston seal, at time = .05 s. The resulting leakage is seen in Fig. 2 and the perturbation in velocity in Fig. 1.

Even though it is theoretically possible to estimate exactly the abovementioned parameters, practical considerations vis-a-vis the selection of eigenvalues for observer performance result in a slight estimation error.

IV. Conclusions

Reduced-order observer theory has been successfully applied to hydromechanical servoactuators. States and a parameter of the actuator have been estimated with minimal estimation error. This could lead to significant developments in fluid power control in that it opens the door to backup provisions, as well as to performance enhancement and failure detection of actuation systems. The measurement system chosen was realistic, and thus successful practical implementation of such a scheme seems almost certain.

References

[1]Leonard, J.B., "A System Look at Electromechanical Actuation for Primary Flight Control," *Proceedings of the IEEE 1983 National Aerospace and Electronics Conference (NAECON)*, pp. 80-86.

[2]Kwa Kernaak, H. and Sivan, R., *Linear Optimal Control Systems*, Wiley Interscience, New York, 1972.

[3]Viersma, T.J., *Analysis Synthesis and Design of Hydraulic Servomechanism and Pipelines*, Elsevier Scientific Publishing Co., 1980.

[4]Merritt, H.E., *Hydraulic Control Systems*, John Wiley & Sons, Inc., New York, 1966.

[5]Gelb, A. (ed.), *Applied Optimal Estimation*, MIT Press, Cambridge, MA, 1974.

[6]Lilly, J.H., "A General Adaptive Observer Structure Subject to Unmodeled Dynamics," *Proceedings of the 1985 American Control Conference*, 1985.

A Computational Method to Solve Nonautonomous Matrix Riccati Equations

M.B. Subrahmanyam*
University of Missouri-Columbia, Columbia, Missouri

Introduction

WE propose a method to solve for the Riccati matrix, which is effective even when the solution of the Riccati equation becomes unbounded at a finite number of points.

Received June 14, 1985; revision received Sept. 24, 1985.

*Assistant Professor, Department of Electrical and Computer Engineering.